Propagation in Broadside-Coupled Suspended-Substrate Stripline in *E*-Plane

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Abstract —**The spectral-domain analysis is applied to the derivation of the propagation characteristics of the even and odd mode, for the broadside-coupled suspended-substrate stripline (BCSSS). The characteristic impedance, based on the current–power definition, as well as the effective permittivity are evaluated. Numerical results are presented illustrating the effects of several different dimensional parameters. Numerical results indicate a large spread between even- and odd-mode impedance for thin substrates ($D/A < 0.045$) and stripwidths ranging $0.046 \leqslant W/B \leqslant 0.47$, suggesting tight coupling. Negligible frequency dependence on odd-mode impedance is evident, as well as significant frequency effects on even-mode impedance. Considerable dispersion is shown to be present in the odd mode for wider strips. Measured results for a low-pass filter and cascaded transitions are presented.**

I. Introduction

PROPAGATION IN *E*-plane circuits (planar circuits suspended in the *E*-plane of a rectangular waveguide) have been studied by a number of authors. Dispersion in finline was first analyzed by Hofmann [1], then followed by others [2]–[8] using hybrid formulations in either the spectral or the space domain. Coupled slots in both unilateral and bilateral finline were studied by Schmidt [10] and Sharma and Hoefer [9].

Numerical results for broadside-coupled suspended-substrate stripline (BCSSS) configured in the *H*- and *E*-planes were published by Allen and Estes [11] using the static capacitance method.

In order to accurately design circuits using this structure, at microwave frequencies, the effect of dispersion should be considered. At present, data on the dispersion, propagation constant, and impedance for BCSSS is scarce in the open literature, although Bornemann [12] has examined the dispersion of the guide wavelength for a similar structure. However, no published data on even-mode impedance is available. This paper will provide numerical data for the effective dielectric constant ϵ_{eff} and impedance in the even and odd propagating modes, taking dispersion into account. Only the dominant EH_0 hybrid mode is considered. In order to demonstrate a practical application of the structure, numerical results are compared to experimental data for a low-pass filter design.

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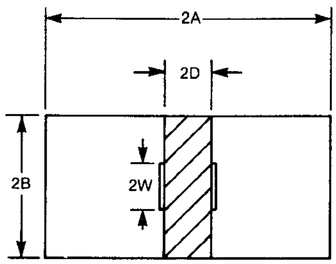

Fig. 1. Broadside-coupled suspended-substrate stripline structure (BCSSS).

II. Analysis

The broadside-coupled line structure, shown in Fig. 1, consists of a thin substrate of relative permittivity ϵ_r suspended in the *E*-plane of a rectangular waveguide with dimensions $2A$ and $2B$. The thickness of the substrate is $2D$ and the strips of width $2W$ are separated by the substrate.

The spectral-domain method, applied to the analysis of the structure, is well documented in other publications [13]–[17]; hence, only a brief outline will be presented. The Fourier transforms of the current densities on the strip conductors and the electric fields in the region of the air–dielectric interface, adjacent to the conductors, are related by the transform of Green's functions as follows:

$$\begin{bmatrix} \tilde{G}_{11}(\alpha_n, \beta, k_0) & \tilde{G}_{12}(\alpha_n, \beta, k_0) \\ \tilde{G}_{21}(\alpha_n, \beta, k_0) & \tilde{G}_{22}(\alpha_n, \beta, k_0) \end{bmatrix} \begin{bmatrix} \tilde{J}_x(\alpha_n) \\ \tilde{J}_z(\alpha_n) \end{bmatrix} = \begin{bmatrix} \tilde{E}_x(\alpha_n) \\ \tilde{E}_z(\alpha_n) \end{bmatrix} \quad (1)$$

where α_n is the Fourier transform variable, β is the propagation constant, and k_0 the free-space wavenumber. $\tilde{J}_x(\alpha_n)$, $\tilde{J}_z(\alpha_n)$, $\tilde{E}_x(\alpha_n)$, and $\tilde{E}_z(\alpha_n)$ are the Fourier transforms of the current densities on the strips and the electric fields adjacent to the strips, respectively.

The current density components on the strips are then expanded in terms of the appropriate basis functions with unknown coefficients. Applying Galerkin's procedure in the Fourier domain along with Parseval's theorem, we obtain a set of algebraic equations in terms of the unknown coefficients [6], [14]. By setting the determinant of the characteristic matrix equal to zero and seeking the root of the resulting equation, a nontrivial solution for the propagation constant and, hence, guide wavelength can be found for each frequency.

The characteristic impedance based on the current–power definition can be computed using [10], [16]

$$Z_{oi} = \frac{2P_{\text{avg},i}}{I_i^2}, \qquad \begin{matrix} i = e, \text{ even mode} \\ i = o, \text{ odd mode} \end{matrix} \tag{2}$$

where I is the longitudinal current on the strips and is given by

$$I = \int_{-w}^{w} J_{z(x,D)}\, dx. \tag{3}$$

The transmitted power P_{avg} is evaluated as follows [18]:

$$P_{\text{avg}} = \frac{1}{2} \operatorname{Re} \int_{-A}^{A} \int_{-B}^{B} \left(E_x H_y^* - E_y H_x^* \right) dx\, dy. \tag{4}$$

In the formulation of the problem presented above, the longitudinal and transverse current densities are expanded in terms of known basis functions with unknown coefficients as follows:

$$\tilde{J}_x(\alpha_n) = \sum_{j=1}^{N} c_j \tilde{\eta}_j(\alpha_n) \tag{5}$$

$$\tilde{J}_z(\alpha_n) = \sum_{i=1}^{M} d_i \tilde{\xi}_i(\alpha_n). \tag{6}$$

The Fourier transforms of the basis functions $\tilde{\eta}_j(\alpha_n)$ and $\tilde{\xi}_i(\alpha_n)$ are evaluated using the discrete transform definition. The unknown coefficients c_j and d_i are those referred to earlier. N and M are the number of expansion functions.

In order to keep the computation simple and efficient, we used only a single basis function for the expansion of each current density component ($N = M = 1$), satisfying the edge condition at $x = \pm W$. The following basis functions were chosen:

$$\eta_1(x) = \begin{cases} x\sqrt{w^2 - x^2}, & |x| < w \\ 0, & \text{elsewhere} \end{cases} \tag{7}$$

$$\xi_1(x) = \begin{cases} \dfrac{1}{\sqrt{w^2 - x^2}}, & |x| < w \\ 0, & \text{elsewhere} \end{cases} \tag{8}$$

The expansion functions described by (7) and (8) are commonly used [6], [9], [15], [17]. For most practical designs, a single term expansion would suffice and should yield sufficiently accurate results [3], [6] with the proper choice of expansion functions. At the same time, computational efficiency is not sacrificed, since only a low determinantal order ($m = 2$) is used. The propagation in a broadside-coupled suspended-substrate stripline can be characterized in terms of two independent modes of excitation, the even and the odd mode. The odd mode corresponds to the case where the current in the two strips flows in opposite directions, while the even mode describes the situation where both currents flow in the same direction.

III. Numerical Results

A computer program was developed which calculates the guide wavelength and even- and odd-mode impedance at each frequency. For the analysis, the propagating modes correspond to the magnetic (even case) and electric (odd case) wall symmetry at $y = 0$, as shown in Figs. 2 and 3, respectively.

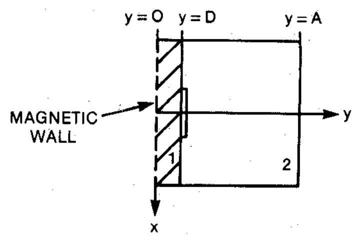

Fig. 2. BCSSS structure showing magnetic wall symmetry (even case).

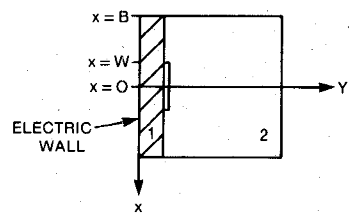

Fig. 3. BCSSS structure showing electric wall symmetry (odd case).

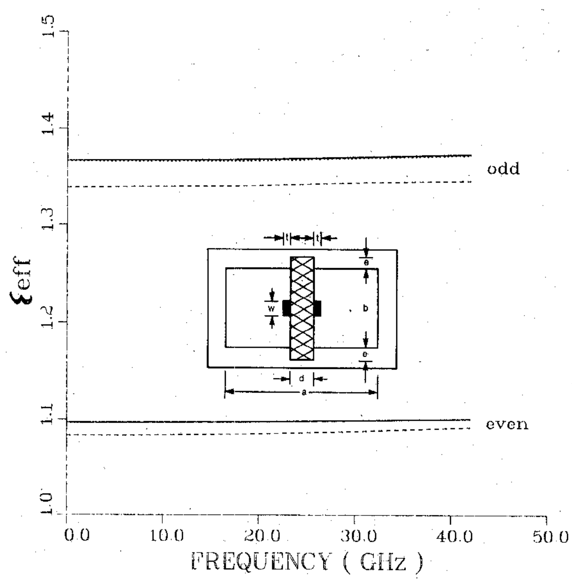

Fig. 4. Even- and odd-mode dispersion for BCSSS structure. $a = 2b = 7.112$ mm, $d = 254$ μm, $w = 0.3$ mm, $\epsilon_r = 2.22$. —— This paper (SDT), $t = e = 0$. - - - Bornemann [12] (modal analysis), $t = 17.5$ μm, $e = 0.5$ mm. ..... GPLINES [3], [8] (SDT) $t = e = 0$.

The accuracy of the analysis and proper functioning of the program was verified by computing the even- and odd-mode effective dielectric constant $\epsilon_{\text{eff}} = (\lambda/\lambda g)^2$ for a BCSSS placed in a WR-28 shield with the substrate, 0.010-in-thick RT/duroid ($\epsilon_r = 2.22$), placed in the center of the broad waveguide dimension. Numerical results, obtained for a normalized stripwidth $W/B = 0.0844$ on substrate thickness $D/A = 0.0357$ and with finite metallization and groove depth effects not taken into account, were compared to those obtained by Bornemann [12] and Jansen [3], [18] shown in Fig. 4.

Agreement with Bornemann is better than 2.1 percent for the odd-mode and 1.2 percent for the even-mode case, over the frequency range indicated. Analytical data in

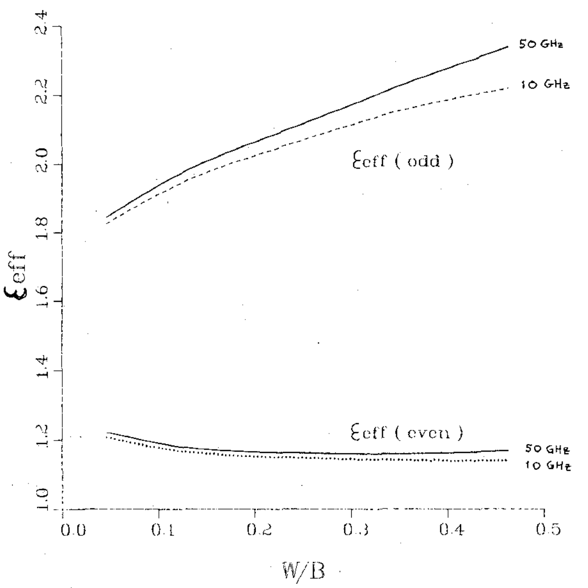

Fig. 5. Propagation characteristics. Effective dielectric constant versus normalized stripwidth W/B, with frequency a parameter. $2A = 10.668$ mm, $2B = 4.318$ mm, $2D = 254$ μm, $\epsilon_r = 2.22$, $t = e = 0$, $0.046 \leqslant W/B \leqslant 0.46$.

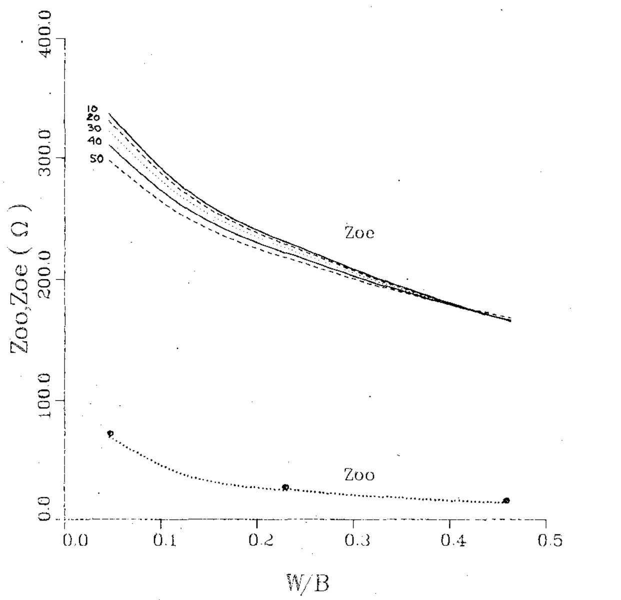

Fig. 6. Impedance characteristics. Z_{oo} and Z_{oe} versus normalized stripwidth W/B, with frequency a parameter. $2A = 10.668$ mm, $2B = 4.318$ mm, $2D = 254$ μm, $\epsilon_r = 2.22$, $t = e = 0$, $0.046 \leqslant W/B \leqslant 0.46$, Z_{oo}. —— This paper. ⊙ SUPER-COMPACT(TM).

Bornemann's thesis were obtained using the method of orthogonal expansion into eigenmodes [20], [21]. Effective permittivity, computed with the spectral-domain technique, tend to be on the order of 2 percent larger than those obtained with the aforementioned analysis [19]. Bornemann's results include the effect of finite metallization thickness and groove depth.

Results obtained in this paper remain valid as long as the thickness of the strips is small compared with all other dimensions. At very high frequencies (E-band), the finite strip thickness would tend to reduce ϵ_{eff} slightly [4], [19]. Neglecting the effect of finite groove depth for the

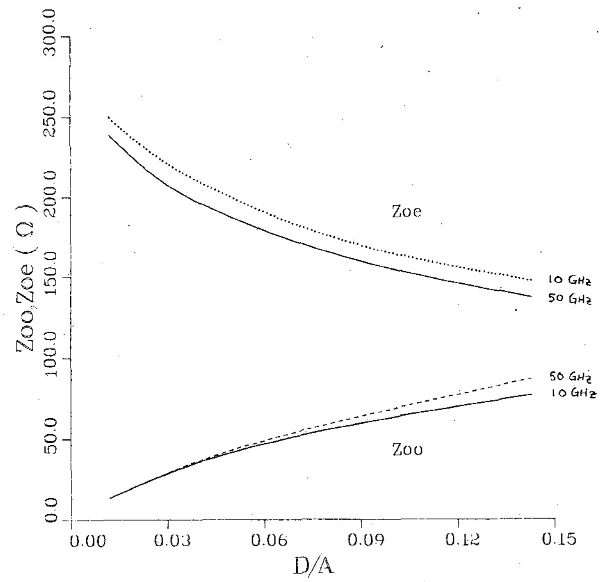

Fig. 7. Impedance characteristics. Z_{oo} and Z_{oe} versus normalized substrate thickness D/A, with frequency a parameter. $2A = 10.668$ mm, $2B = 4.318$ mm, $W/B = 0.2316$, $0.012 \leqslant D/A \leqslant 0.143$, $\epsilon_r = 2.22$, $t = e = 0$.

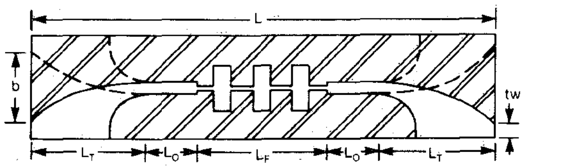

Fig. 8. Low-pass filter in broadside-coupled suspended-substrate stripline (BCSSS). $L_T = 19.05$ mm, $L_0 = 15.629$ mm, $L_F = 6.842$ mm, $L = 76.2$ mm, $b = 4.318$ mm, $\epsilon_r = 2.22$, $t_w = 2.06$ mm (wall thickness of open-type split-block housing).

frequency range used is not thought to significantly alter the results [19]. Since the odd-mode case is electrically identical to the well-known shielded microstrip line, a comparison was made for ϵ_{eff} with data obtained from the program GPLINES [18]. The spectral-domain technique used in this program [3] uses a four-term expansion with sinusoidal basis functions which are modified by an edge condition term so as to closely approximate the strip current. Agreement with Jansen is within 0.1 percent.

Analytical results computed for a BCSSS structure, placed in a WR-42 shield and printed on 0.010-in RT/Duroid 5880 ($\epsilon_r = 2.22$), are shown in Figs. 5–7.

Plotted in Fig. 5 is the even- and odd-mode effective dielectric constant as a function of normalized stripwidth W/B, for $D/A = 0.0238$, with frequency as a parameter. Holding W/B fixed, the odd mode ϵ_{eff} increases significantly with frequency for wider strips ($W/B > 0.2$). Dispersion for this mode is 5.7 percent for $W/B = 0.46$ and 1.0 percent for $W/B = 0.046$. When W/B is increased for a given frequency, ϵ_{eff} rises sharply. In the even mode, dispersion does not appear to be as significant as in the odd mode. Less than 2.6-percent dispersion was computed for wide strips ($W/B = 0.46$) and 1.4 percent for narrow strips ($W/B = 0.046$). Changes in stripwidth do not appear to affect the effective dielectric constant significantly for $0.15 \leqslant W/B \leqslant 0.46$.

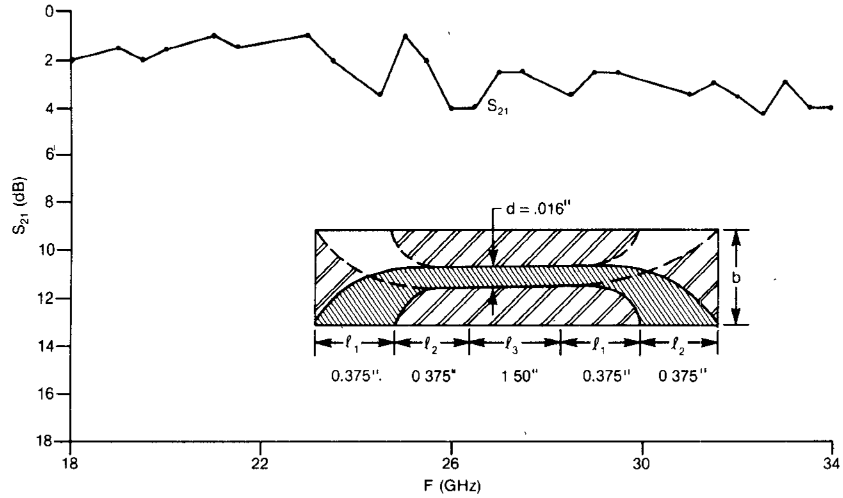

Fig. 9. Insertion loss measurement for a BCSSS back-to-back transition, including 1.50-in line section. Substrate is 0.010-in (254-μm) RT/Duroid 5880. Tapers are single exponential. --- Back-side metallization. // // Substrate. —— Front-side metallization.

Fig. 6 illustrates the behavior of the even-mode impedance as a function of W/B for several frequencies. A definite frequency dependence is observed for $W/B < 0.35$, with 11.7-percent change in Z_{oe} for narrow strips and 2 percent for wide strips. The odd-mode impedance is practically frequency independent, with less than 1.2-percent impedance change between 10 GHz and 50 GHz over the complete W/B range. Odd-mode impedance results in this paper were compared with those obtained with SUPER-COMPACT's(TM) closed-form expressions for covered microstrip line, which uses Wheeler's expressions [22] and Getsinger's dispersion model [23]. Agreement is good, with better than 0.5 percent for wide strips.

The effect of substrate thickness on even- and odd-mode impedance is illustrated in Fig. 7. As is evident, a large spread exists betwen Z_{oo} and Z_{oe} for thin substrates with $D/A < 0.045$, enabling one to obtain tight coupling. The results are computed for $W/B = 0.2316$. As previously noted, the even-mode impedance is noticeably frequency dependent, while the odd-mode impedance is not, for thin substrates $(D/A < 0.045)$. Coupling is adjusted with stripwidth $2W$ and substrate thickness $2D$.

IV. Experimental Results

Using the computed results described in this paper, a low-pass filter was designed in broadside-coupled suspended-substrate stripline, shown in Fig. 8.

In order to achieve the BCSSS planar circuit, an antipodal transition between rectangular waveguide and the filter structure is required. To evaluate the attenuation associated with such a transition, a back-to-back antipodal transition was fabricated, as shown in Fig. 9. Also illustrated in this figure is the insertion loss of such transition using an 0.010-in-thick RT/Duroid 5880 substrate, placed in a WR-42 split block housing. The design frequency was 28.5 GHz. The large ripple in S_{21} was found to be due to reflections at the antipodal transitions on both ends of the coupled line section. A computer-controlled automatic network analyzer was used to obtain the results.

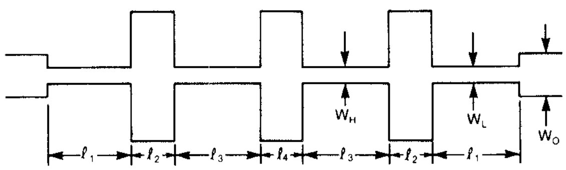

Fig. 10. Dimensions of low/high impedance sections. $l_1 = 0.933$ mm, $l_2 = 0.540$ mm, $l_3 = 1.645$ mm, $l_4 = 0.606$ mm, $W_0 = 0.4$ mm, $W_L = 2.00$ mm, $W_H = 0.2$ mm.

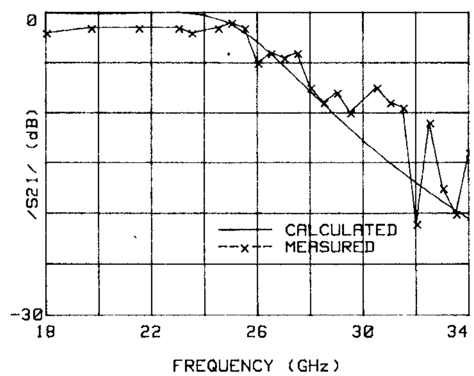

Fig. 11. Magnitude of S_{21} parameters for low-pass filter.

A. Measurements on the Low-Pass Filter

A seven-section low-pass filter, shown in Figs. 8 and 10, was developed and tested. The low and high odd-mode impedance sections were chosen to be 14 Ω and 70 Ω, respectively, in a 47-Ω system. Design frequency is 25 GHz. Substrate material is RT/Duroid 5880, 0.010-in-thick, placed in the center in the E-plane of a WR-42 housing. Design criteria were $f_{c0} = 25$ GHz and in-band ripple $\alpha_r = 0.01$ dB. Measured S_{21} and S_{11} results track the computed response very well, as shown in Figs. 11 and 12. The discrepancy between predicted and measured insertion loss, in Fig. 11, is due to the 2-dB loss in the two transitions (Fig. 9). Some large ripple in S_{21} outside the pass-

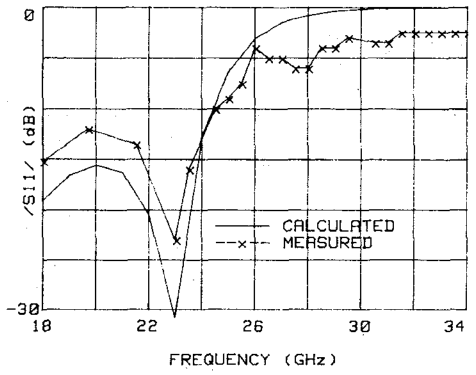

Fig. 12. Magnitude of S_{11} parameters for low-pass filter.

band is evident, but this may be due to poor performance of the antipodal transition at higher frequencies.

V. Conclusion

Even- and odd-mode impedance and propagation constants for a broadside-coupled suspended-substrate stripline have been computed using the spectral-domain technique. The even-and odd-mode effective dielectric constant and odd-mode impedance computed in this paper were found to give good agreement with previously published results.

Numerical results indicate significant dispersion for wide strips ($W/B > 0.2$) and little frequency dependence of the impedance in the odd mode. In the even mode, frequency dependence of impedance is noticeable for $W/B < 0.35$, while dispersion effects are negligible over most practical stripwidths ($0.1 < W/B < 0.46$). The response of a low-pass filter was found to agree reasonably well with predicted values of insertion and return loss.

Acknowledgment

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